Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 4**

A bar is free to move in the horizontal direction as shown. Determine the angular velocities of the free vibrations and the corresponding modes. Use one bar element of nodes 1 and 2. Cross-sectional are *A*, density of the material, and Young’s modulus  of the material are constants.

*L*

*x,X*

2

1

**

**Solution template**

The non-zero displacement/rotation components of the structure are  and . Let us start with the element contributions for the internal and inertia parts

, .

As the axes of the material and structural coordinate systems coincide, virtual work expression of the structure takes the form

.

Principle of virtual work and fundamental lemma of variation calculus imply the set of ordinary differential equations

.🡸

The angular speeds of free vibrations can be deduced from the stiffness and mass matrix of the equation system

 and   .

The angular speeds of free vibrations are the eigenvalues of . Let us start with the eigenvalues of  to get

 

 and .

The corresponding modes follow from the linear equation system of the eigenvalue problem when the eigenvalues are substituted there (one by one)

 and   ,

 and   .

As , the angular velocities of the free vibrations and the associated modes are

 and . 🡸